AP Calculus AB Syllabus

Curricular Requirements

CR1 The students and teacher have access to a college-level calculus textbook, in print or electronic format.

See pages: 3, 4

CR2 The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.

See pages: 2, 6

CR3 The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes.

See pages: 6, 7, 9, 11, 13, 14

CR4 The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations.

See pages: 2, 6, 7, 10, 14, 16, 20, 22

CR5 The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification.

See pages: 7, 8, 13, 15

CR6 The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation.

See pages: 2, 7, 9, 12, 13, 19

CR7 Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.

See pages: 3, 8, 9, 13, 19, 23, 26

CR8 The course provides opportunities for students to use calculus to solve real- world problems.

See pages: 6, 8, 13, 27

Course Overview

Course Overview: AP® Calculus AB is equivalent to a first-semester college calculus course. Topics include functions, limits and continuity, derivatives, and integrals. The course will focus on applying the skills and concepts of calculus to modeling and solving problems across multiple representations.

The course is aligned with the **College Board AP Calculus AB Course and Exam Description** and is equivalent to **one semester of college-level calculus** (CR1). Students are required to take the AP Calculus AB Exam in May.

Course Expectations

Students are expected to complete all homework problems to the best of their ability. If they need additional support, they can refer to the additional resources listed below.

The Personal Progress Checks (PPC) that are assigned online for this course through the student's College Board account are to be completed on time.

Students will take weekly quizzes. These quizzes are short and are intended to check for understanding of concepts and skills that were recently taught. Students are required to make all corrections when the quizzes are returned to them. Students work with functions represented:

- graphically
- numerically
- analytically
- verbally

with emphasis on connecting these representations and communicating mathematical reasoning clearly **(CR4, CR6).**

The course is organized around the Big Ideas:

Big Idea #1: Change
Big Idea #2: Limits
Big Idea #3: Analysis of Functions (CR2)

Course Outline and Pacing:

- September/October – Unit 1
- October/November – Units 2 and 3
- November/December – Unit 4
- December/January – Unit 5
- January/February – Unit 6
- February/March – Unit 7
- March/April – Unit 8
- April/May – AP Review

Graphing Calculator and Technology (CR7)

Students are required to have individual access to an approved graphing calculator such as a **TI-83/84 (or equivalent)**.

- Students are exposed to numerous calculus applets during the course.
- Students download a number of calculator programs from the Resources Page online, including programs for Riemann Sums, Area between two curves, and Slope Fields. These programs are designed to help students visualize the various concepts and to get a deeper understanding of calculus.
- Students are instructed throughout the course of the Four Functionalities allowed on the AP Exam with the graphing calculator including:
  - Plot the graph of a function within an arbitrary viewing window.
  - Find the zeros of functions (solve equations numerically).
  - Numerically calculate the derivative of a function.
  - Numerically calculate the value of a definite integral.
  - Explore or interpret calculus concepts.
- I instruct students on the various software packages to illustrate volumes of solids, slope fields, and accumulation.
- During the course, problems will be represented and solved in four distinct ways: analytically, numerically, graphically, and verbally. Students will use a graphing calculator to determine the value of various limits, to determine the value of a derivative at a point, to find the value of a definite integral, to graph a function in various windows, and to solve a variety of equations, as well as explore concepts such as the limit of a function at a point.

Example:
Students compare numerical and analytical derivatives to understand approximation and error.

Textbooks, References, and Materials (CR1)

Primary Textbook: Larson, Ron, and Battaglia, Paul. Enhanced WebAssign with *Calculus for AP.* 2nd ed. Boston: Cengage Learning, 2021.

AP Review Text: Howell, Mark, and Martha Montgomery. *Be Prepared for the AP Calculus Exam.* Andover, MA: Skylight Publishing, 2016.

Reference Books:

Hughes-Hallett, Deborah, Andrew M. Gleason, et al. *Calculus Single Variable*. 4th ed. New York: John Wiley & Sons, Inc., 2005.

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus: Graphical, Numerical, Algebraic.* 4th ed. Upper Saddle River, NJ: Prentice Hall, 2012.

Foerster, Paul A. *Calculus: Concepts and Applications*. 2nd ed. Emeryville, CA: Key Curriculum Press, 2005.

Lederman, David. *Multiple-Choice & Free Response Questions in Preparation for the AP Calculus (AB) Examination*. 10th ed. New York: D & S Marketing Systems, Inc., 2016.

McMullin, Lin. *Teaching AP Calculus*. 3rd ed. Brooklyn, NY: D & S Marketing Systems, 2015.

Best, George, and Sally Fischbeck. *AP Calculus with the TI 83 Graphing Calculator*. Andover, Mass.: Venture Publications, 1998.

Software:

Best, George. *Best Grapher*.

Bradford, William. *Calculus AB Test Bank*.

Desmos

Weeks, Audrey. *Calculus in Motion*.

- Previously Published AP Multiple-Choice and Free-Response Questions including the 1997, 1998, 2003, 2008, 2012 released AP Exams
- AP Professional Development Workshops and Institute materials
- AP Central® website and AP Calculus OTC
- TI-83+ and TI-84 graphing calculators

Students also use AP Classroom topic questions and Personal Progress Checks as part of regular practice and review.(**CR1**)

Additional Resources

- Students can watch a video on Web Assign provided by Cengage eBook Calculus for AP, our textbook, corresponding to the lesson we covered in class. On a regular basis, I send an announcement to remind students of this resource.
- Students can log in to the website "GetAFive" using the instruction sheet provided. This site has videos and problems grouped according to topic.
- Students have the option of coming to me for help during my office hours.

Student Practice

Throughout each unit, Topic Questions will be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during a lesson, or as homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, Personal Progress Checks will be provided as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart their progress, and their results will come with rationales that explain every question's answer. A class period is set aside to re-teach skills based on the results of the Personal Progress Checks.

An extra day each week is devoted to an appropriate calculator activity, multistep word problems, Topic Questions, Personal Progress Checks, and/or free response questions (FRQ's) from released AP Calculus BC Exams. Emphasis is placed on problem solving, using the calculus in new settings, and helping students to see the connections among the big ideas and the major themes in calculus. FRQs, which emphasize real-world applications of the calculus are selected for discussion on the Discussion Board on this extra day.

Course Delivery and Student Interaction

The course is also designed around the four Mathematical Practices in AP Calculus outlined in the 2020 CED including:

Practice #1: Implementing Mathematical Processes
Practice #2: Connecting Representations
Practice #3: Justification
Practice #4: Communication and Notation

This course is delivered online. Students access lectures, assignments, and resources through the course website. Discussion-based work using the Discussion Board is a regular part of the course. Students:

- post complete solutions
- ask questions
- challenge reasoning
- explain concepts using correct notation

These interactions develop the four Mathematical Practices:

- justification
- communication and notation
- implementation of processes
- connecting representations

Course Outline and Description: CR2

Unit 1: Limits and Continuity (Big Ideas: Change, Limits, Analysis of Functions)

1.1 Introducing Calculus: Can Change Occur at an Instant? (Skill 2.B)

In an activity, students will calculate the velocities (the average rate of change) of several automobiles using both functions given analytically and data presented in a table of time versus displacement. Students will use their information to approximate the instantaneous velocity of the automobile at a particular time t and to sketch a graph of velocity as a function of time. They will provide a verbal (that is, written in words) interpretation of the movement of each vehicle (such as "The car's velocity is positive and decreasing") and explain how their verbal interpretation is connected to the graph they have drawn. CR4 CR8

1.2 Defining Limits and Using Limit Notation (Skill 2.B)

In an activity, students will match a list pertaining to the graph of a function consisting of vertical asymptotes, horizontal asymptotes, jump, removable, and non- removable discontinuities. Students will have to match selected portions of the graph to its written description and symbolic (notation) description. Here, students are learning how to express limits in both written and symbolic form to understand the behavior of a function f as f gets sufficiently close to a particular x-value.

1.3 Estimating Limit Values from Graphs (Skill 2.B)

In an activity, students will use a graph of a function to approximate the value of a limit, if it exists. Students will use the strategy of Concepts with Color, located on page 204 in the CED, where the student will trace the graph of the function from the left in one color and will trace the graph from the right using another colored pencil. Then, using correct language to describe a limit, students will explain whether or not the limit exists.

1.4 Estimating Limit Values from Tables (Skill 2.B)

In a homework assignment, students will complete a table of values to find the limit, if it exists, for a set of functions. In some of the problems, a graphing calculator will be required. Students may notice in some problems that direct substitution would have worked, while in other problems, direct substitution does not work, but the problem still has a limit. A problem where direct substitution fails but still has a limit gets the student to think about how else they could come up with the answer without using technology. (Getting them prepared to think about using algebra.)

1.5 Determining Limits Using Algebraic Properties of Limits (Skill 1.E)

Students will complete a homework assignment applying the Algebraic Properties of Limits across multiple representations. Students will be given information about the graph of function f, a polynomial function g expressed symbolically, a rational function h expressed symbolically, a table of values for a function k, and a written description of the limits for functions r and s. Although all functions may not be used in one problem, each limit problem will consist of at least two different representations, and students will be asked to explain how those representations are connected. In addition to finding limits across multiple representations, students will discover in a problem or two that although the limit of a function f and the limit of a function g may not exist, the limit of $f + g$, does exist. CR4

1.6 Determining Limits Using Algebraic Manipulation (Skill 1.C)

Students will complete a homework assignment where they are given limits of various functions expressed analytically. The students will have to identify the appropriate mathematical procedure (including direct substitution, factoring, finding a common denominator, multiplying by a conjugate, and rewriting the expression) and then implement that procedure to compute the limit. CR3

1.7 Selecting Procedures for Determining Limits (Skill 1.C)

Students will complete an activity where they have to choose a method for determining a limit arranged in a chart. They will start with direct substitution; if they get 0/0, they will have to choose from Algebra, Table of Values, or a Graph as a means for finding the limit. Then, they will write a brief explanation why they chose that method for finding the limit. Students will also use a flow chart to help them find limits. CR3

- ☐ Complete Personal Progress Check MCQ Part A for Unit 1

1.8 Determine Limits Using the Squeeze Theorem (Skill 3.C)

Students will complete a three-part homework assignment using the Squeeze Theorem. For each part, students will have to decide if the conditions of the Squeeze Theorem are met and, if so, provide the reasoning for their claim that the conditions are satisfied and then proceed to use the theorem to find the indicated limit. The first part of the worksheet will consist of graphs where students have to decide if the conditions are met. In the second part, a function will be sandwiched between two other functions and students will have to check if the conditions are met before finding the limit. In the third part, the students will be given a function where they will have to sandwich the function between two values and proceed from there in trying to find the limit. CR5

1.9 Connecting Multiple Representations of Limits (Skill 2.C)

Students will complete a homework assignment to review the limits they've studied so far. This assignment will be broken into parts: in part one, students will use a graph to find the limit; in part two, students will use a table of values to find the limit; in part three, students will use algebra to find the limit. In the final part, students will have to use multiple representations to find a limit. The representations will include two graphs, two functions, and a table of values. CR4

1.10 Exploring Types of Discontinuities (Skill 3.B)

Students will complete an activity in class where they will learn the different types of discontinuities. In one part, students will complete a chart using the given graph of a function. The columns of the chart will consist of finding

$$f(a),\ \lim_{x\to a+} f(x),\ \lim_{x\to a-} f(x),\ \lim_{x\to a} f(x),\ f(a)= \lim_{x\to a} f(x).$$

The students will learn about three types of discontinuities by completing this table – removable, jump (piecewise), and asymptotic. They will also justify the type of discontinuity using correct notation. We will also refer back to the activity in Topic 1.2. CR6

1.11 Defining Continuity at a Point (Skill 3.C)

After students complete the activity from 1.10, they will learn what conditions are required for a function to be continuous at a point. Students have a tendency to give weak explanations for justifying whether a function is continuous at a point or not. They fail to use proper notation and need practice applying the definition of continuity to problems in a variety of representations. Also, to help students achieve better communication and notational fluency with the definition of continuity, I will use an activity that includes error analysis. Students will critique posted student samples from prior FRQ's that either correctly or incorrectly used the definition of continuity in justifying answers. We will also refer back to the activity in Topic 1.2. CR5

- ☐ Complete Personal Progress Check MCQ Part B for Unit 1

1.12 Confirming Continuity over an Interval (Skill 1.E)

Students will complete a homework assignment where they have to check for continuity over different types of intervals, i.e., closed, open, half-open, etc. Problems will consist of functions that are not continuous at an interior point of an interval, endpoint of an interval, and at some point where no interval is given. Problems will also consist of functions that are continuous on the given interval. Piecewise functions will be emphasized in this assignment because students fail to check for continuity where the domain is broken up. Confirming continuity is an essential condition for Existence Theorems.

1.13 Removing Discontinuities (Skill 1.E)

Students will complete a homework assignment consisting of problems where a function is not continuous at a point, but the problem can be rewritten or extended so that the function is now continuous at that point.

1.14 Connecting Infinite Limits and Vertical Asymptotes (Skill 3.D)

Using a table of values for x, students will use a calculator to find values for a given function $f(x)$. They will notice that the values for $f(x)$ either approach positive or negative infinity. Then students will use their graphing calculator to explore the graph of the function so that they can verify the location of the vertical asymptote. Using the table of values, students will use limit notation to explain why the function has a vertical asymptote near that value of x. We will also refer back to the activity in Topic 1.2. CR7

1.15 Connecting Limits at Infinity and Horizontal Asymptotes (Skill 2.D)

Students will complete an activity broken into three parts. In the first part, students will indicate what the y-values of a function are approaching as the x - values approach positive or negative infinity. In the second part, students will use technology to graph a given function and use their graph to determine the equation of the horizontal asymptote. In the third part, students will determine the horizontal asymptotes without technology by using the information they obtained in parts one and two. Students will have to make the connection in parts one and two in order to answer part three without technology. However, students may use technology to confirm the horizontal asymptotes of a function in part three. We will also refer back to the activity in Topic 1.2. CR8

- ☐ Complete Personal Progress Check FRQ A for Unit 1

1.16 Working with the Intermediate Value Theorem (Skill 3.E)

Students will complete an activity using the Intermediate Value Theorem. In order to apply the IVT, students must address the essential condition of continuity on a closed interval. In part one, students will use the strategy of sentence starters, indicated in the CED on page 212, to check for continuity on a closed interval. In part two, students will use a template to write an argument using IVT. The problems in part two will include a variety of contexts in which students have to apply IVT. CR5

- ☐ Take Unit 1 Test.

Unit 2: Differentiation: Definition and Fundamental Properties (Big Ideas: Change, Limits, Analysis of Functions)

2.1 Defining Average and Instantaneous Rates of Change at a Point (Skill 2.B)

Students will use the graph of a function to find the average rate of change (the slope of the secant line) of a function over several closed intervals. Then, students will approximate the instantaneous rate of change at a point (the slope of the tangent line) using their average rates of change. The activity will consist of several representations from past FRQ’s consisting of tables of values, graphs, and quantities modeled by a function. Units will be required.

2.2 Defining the Derivative of a Function and Using Derivative Notations (Skills 1.D and 4.C)

Using the activity from 2.1 as a reference, students will learn the definition of derivative in three different forms.

In a homework assignment, students will use all three forms to find the derivative of a function. On this same assignment, students will be given limits of various difference quotients with the denominator approaching 0 and will be asked to identify each as the derivative of a specific function at a specific x-value; for each limit, the students will write in appropriate mathematical language an explanation of how they determine what derivative the limit represents. CR6

2.3 Estimating Derivatives of a Function at a Point (Skill 1.E)

In a Zoom activity, students will estimate the derivative of a function at a point using a table of values.

The class activity will consist of several representations from past FRQ's where students will need to show a difference quotient and attach units to their answer.

2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist (Skill 3.E)

In a homework activity, students will learn to determine if a function is differentiable or not. At this point in the course, students interpret a function being differentiable at a point $x = a$ as the slope of the tangent line existing at $x = a$. In the first part of the activity, students will be given the graphs of two functions; one graph is of a continuous function, and the other graph is a function that is not continuous at a given point. Students will draw several tangent lines along the graphs of both functions using a straight edge. Then, students will use the Discussion Board to discuss which functions are differentiable and why. They will discuss the essential condition for differentiability. After students learn that continuity is a requirement for differentiability, they will complete the second part of the activity, where they will consider graphs of continuous functions and draw tangent lines at various points along their graphs. The students should discover that slopes of tangents do not exist at corner points, cusps, or vertical lines. CR6

2.5 Applying the Power Rule (Skill 1.E)

In a class activity, students will use their graphing calculator to discover the power rule for derivatives. Students will enter functions such as, $y = x$, $y = x^2$, $y = x^3$ into their calculators and graph the derivatives of the functions one at a time in order to explore the graphs and make a conjecture about the derivative of a power function. Then, students will use the Discussion Board to try and generalize a rule for finding the derivative of a power function. CR7

☐ Complete Personal Progress Check MCQ A for Unit 2

2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple (Skill 1.E)

Students will complete a homework assignment applying the power rule to equations of the form x^n. In numerous problems, students will have to perform algebra first essentially rewriting functions involving products and quotients using algebra, before applying the power rule. CR3

2.7 Derivatives of cos x, sin x, e^x, and ln x (Skill 1.E)

In several class activities, students will discover the derivative rules for these functions in multiple ways. First, in a Zoom class, we will derive the derivative of cos x using the definition of derivative. Second, students will graph one period of the sine curve and then draw various tangent lines at selected values of x to sketch the derivative. Third, students will explore the derivatives of e^x and ln x by using their graphing calculator. Finally, students will have to recognize the limit expression as the definition of derivative. CR4

2.8 The Product Rule

In a homework assignment, students will use the product rule to find the derivative involving the functions listed in the topics above. The assignment will consist of the following parts. In part one, students will find the derivative given the equation of a function. In this part, the derivative of some problems should use algebra first and would not necessarily warrant a product rule even though the function is a product. In part two, students will use a table of values to find the value of the derivative. Students will have to pull values from the table in order to compute their answer. In the third part, students will use the graphs of functions to find the value of the derivative. (Skill 1.E)

2.9 The Quotient Rule (Skill 1.E)

In a homework assignment, students will use the quotient rule to find the derivative involving the functions listed in the topics above. The assignment will be cumulative involving all the other derivative rules, but an emphasis will be on the quotient rule. In part one, students will find the derivative given a symbolic representation of a function. In this part, the derivative of some problems should use algebra first and would not necessarily warrant a quotient rule even though the function is a quotient. In part two, students will use a table of values to find the value of the derivative. Students will have to pull values from the table in order to compute their answer. In the third part, students will use the graphs of functions to find the value of the derivative.

- ☐ Complete Personal Progress Check FRQ A for Unit 2

2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions (Skill 1.D)

In a homework assignment, students will apply the derivative rules of Tangent, Cotangent, Secant, and Cosecant Functions. The derivatives in this homework assignment will be cumulative of all the other derivative rules, but an emphasis will be placed on the new rules.

- ☐ Complete Personal Progress Checks MCQ B and FRQ B for Unit 2
- ☐ Take Unit 2 Test

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions (Big Idea: Analysis of Functions)

3.1 The Chain Rule (Skill 1.C)

In a homework assignment, students will discuss how they would solve Free-Response Question 3 – Part C from 2007 (Form B). Students will have to pay attention to the fact that they need to find the derivative $w'(t)$ when they are not given a formula that directly relates w and t; the formula given in the stem of the problem is in terms of w and r. This kind of problem should prompt a good discussion on how to find this derivative. To make things a little less complicated, I will write on the board a pair of equations like $y = u^3$ and $u = x^2 + 1$ and ask the students to find $\frac{dy}{dx}$. I don't expect students to come up with this rule on their own, so I will demonstrate $\frac{dy}{dx}$ for the students on how we could write a chain rule (a chain of derivatives) to find $\frac{dy}{dx}$ Second, I will now denote $u = g(x) = x$ and ask students to find $f(g(x))$ and ask them how we could take the derivative of $f(g(x))$ and end up with our answer from before. This will now lead to the derivative of $f(g(x))$ using $f'(g(x))g'(x)$. After demonstrating a few examples, I will return to FRQ 3 and ask the students now to write a chain rule with the information they just learned. I will emphasize that students should pay attention to the units and use the units as a guide in writing a valid chain rule.

In a homework assignment, students will apply the chain rule in a variety of situations. In part one, students will be given a pair or more of equations and will have to write the chain rule using Leibniz notation to find the derivative. In part two, students will find the derivative of $f(g(x))$ without writing the chain rule. In part three, students will be given word problems like the FRQ they just worked with where they will have to write a chain rule to find the derivative paying attention to the context and units of the problem. CR3

3.2 Implicit Differentiation (Skill 1.E)

In a homework assignment, students will use implicit differentiation to find the derivative. In addition to performing implicit differentiation, students will need to find points along a curve where the tangent line is vertical or horizontal.

3.3 Differentiating Inverse Functions (Skill 3.G)

In a homework assignment (the day prior to this actual lesson), students will recall the properties of inverse functions. Students will be guided in steps to first find the inverse of a linear function, confirm algebraically that they obtained the correct inverse function by using $f(g(x)) = x = g(f(x))$, and confirm graphically that they obtained the correct inverse function by graphing, f, g, and $y = x$ on one set of axes and noting the symmetry of f and g about the line $y = x$. In the final part of this assignment, students will find the derivatives of f and g and be asked how their slopes compare. The next day in a homework assignment, students will derive a formula for finding the derivative of inverse functions by finding the derivative of the equation $f(g(x)) = x$. Once the rule is established for finding derivatives of inverse functions, students will practice the notation for this rule using different pairs of functions. This is helpful because students often struggle with notational fluency here. CR6

3.4 Differentiating Inverse Trigonometric Functions (Skill 1.E)

Students will apply implicit differentiation to trigonometric inverse relations like sin $y = x$ to generate a rule for finding the derivative of $sin^{-1}(x)$ as well as the remaining trigonometric functions.

- ☐ Complete Personal Progress Check FRQ B for Unit 3

3.5 Selecting Procedures for Calculating Derivatives (Skill 1.C)

In a homework assignment, students will be challenged to use the flowchart from ***Teaching and Assessing Module 2*** to determine which derivative rule to apply to a given function. The functions are represented as $f(x)$, y, etc. One student will propose how to select the derivative procedure on the Discussion Board while the other students explain why they agree or disagree with the procedure chosen. Students then find the derivative using appropriate symbols for the derivative. They check their answers and notation. Other students can volunteer to switch roles after each problem. CR6

3.6 Calculating Higher-Order Derivatives (Skill 1.E)

In a Zoom Break Out Session activity, students will work in groups of four. Each group will be given four derivative problems on index cards. For each problem, there will be three specific derivatives to find while the fourth derivative will be a general rule for finding the nth derivative of the original problem. The first three students within each group will find the indicated derivative while the fourth person will find a general rule in terms of n for finding the nth derivative. Each person in a group gets one turn at finding a rule for the nth derivative.

- ☐ Complete Personal Progress Checks MCQ and FRQ A for Unit 3
- ☐ Take Unit 3 Test

Unit 4: Contextual Applications of Differentiation and Rates of Change (Big Ideas: Change, Limits)

4.1 Interpreting the Meaning of the Derivative in Context (Skill 1.D)

In a class activity, students will start with a function $G(x)$ without context and be asked to interpret $G'(5)$. Then, context will be added to function $G(t)$ to mean the amount of unprocessed gravel arriving at a processing plant, where G is measured in tons and t is measured in hours. Since students often struggle interpreting a derivative, students will be provided with a template. The template will be: At time t = _____________, the function is increasing or decreasing at a rate of _______________(units of y)/(units of x). This activity will require students to interpret different representations. In a final part of this activity, students will critique student samples from past free-response questions to learn both correct and incorrect ways of interpreting a derivative. CR8

4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration (Skill 1.E)

Students will recall that average velocity is the change in position divided by the change in time, i.e., average velocity = $\frac{s(t+h)-s(t)}{h}$ and that when we take $\lim_{h\to 0}\frac{s(t+h)-s(t)}{h}$ get $s'(t)$, which refers to the instantaneous velocity at time t. Students will also recall that average acceleration is the change in velocity divided by the change in time, i.e., average acceleration is $\frac{v(t+h)-v(t)}{h}$ and when taking the $\lim_{h\to 0}\frac{v(t+h)-v(t)}{h}=v'(t)=a(t)$

Then, using guided examples, students will solve motion problems finding when the particle is at rest, reverses direction, speeding up, moves right, moves left, speeds up, and slows down using function and graphical representations. Students will have to give reasons for their answers. CR3 CR5 CR6

4.3 Rates of Change in Applied Contexts Other Than Motion (Skill 2.A)

In a homework assignment, students will find rates of change with respect to quantities other than time. Former free- response questions will be included where students will find the value of a derivative using their calculators and interpret their answers in the context of the problem. Questions will vary in their representation. Some questions will be represented in symbolic form, e.g., given $S(h)$ find $S'(4)$, while other questions will be in the form of words, e.g., "If h is the vertical distance between the graphs of functions f and g, then find the rate at which h changes with respect to x when $x = 1.8$." CR7

4.4 Introduction to Related Rates (Skill 1.E)

Students often struggle with Related Rates problems for numerous reasons. Therefore, before solving Related Rate problems, students will use a class activity to help them develop guidelines for solving related rate problems. The steps we will focus on during this activity consist of: 1. Drawing a picture, if applicable, to represent the problem; 2. Distinguishing between quantities that change and those quantities that don't change and labeling those quantities that change a variable; 3. Writing an equation that relates the quantities in the problem; and 4. Practice differentiating quantities in the equation with respect to time.

In this step, students will need to identify the appropriate rule for differentiation based on the classification of the expression. CR3

4.5 Solving Related Rate Problems (Skill 3.F)

This lesson will continue from the topic 4.4 listed above. In a Zoom break out session, students will work in pairs. One student will draw a picture and label the quantities that change as variables and then pass it on to their partner for verification. Once the pair agrees on the picture and labels, they will individually write an equation that relates the quantities in the problem and switch papers to see if they agree with each other's equations. Then, the students will differentiate the equation together agreeing on their steps in finally solving the problem. CR4

4.6 Approximating Values of a Function Using Local Linearity and Linearization (Skill 1.F)

This class activity will begin with me posting a challenge on the Discussion Board asking students to guess $\sqrt[6]{6}$ to 3 decimal places. The activity will continue with the steps in the handout. At the end of the activity, the student who is the closest wins a prize.

- ☐ Complete Personal Progress Check FRQ A for Unit 4

4.7 Using L' Hospital's Rule for Determining Limits of Indeterminate Forms (Skill 3.D)

Students will complete an activity using their calculators to enhance their understanding of L' Hospital's Rule. The first part of the homework will require students to find the limit using both algebra and L'Hospital's Rule. Students should realize that even though both methods are capable of producing the same result, L' Hospital's Rule does have an advantage in certain problems. Limit problems in the homework will also represent the definition of derivative. In this case, the take-away is that students should realize that they could evaluate a limit representing the definition of derivative using L' Hospital's Rule.

- ☐ Complete Personal Progress Checks MCQ and FRQ B for Unit 4
- ☐ Take Unit 4 Test

Unit 5: Analytical Applications of Differentiation including Analysis of Functions (Big Idea: Analysis of Functions)

5.1 Using the Mean Value Theorem (Skill 3.E)

Students will use the graphic organizer and template from ***Teaching and Assessing Module 3.*** The graphic organizer will help students to check for the conditions of continuity on a closed interval and differentiability on an open interval. Then, students will use the template as a strategy to write a mathematical argument. These strategies are helpful because students often have a difficult time addressing the conditions of MVT and putting all the pieces together to write an argument.

In a homework assignment, students will be expected to apply MVT in multiple representations. The problems will be represented by using functions, graphs, and tables. For each representation, students will have to explain whether or not MVT can be applied. If MVT can be applied, then students will write an argument using the template mentioned above to justify their answer. CR5

5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points (Skill 3.E)

Students will consider several graphs of functions – continuous and discontinuous. Using the graphs, students will take away that if a function is continuous on a closed interval, then the function has both a maximum and minimum. Students will understand that a function may still have a maximum or minimum even though it is not continuous on a closed interval. CR5

5.3 Determining Intervals on Which a Function is Increasing or Decreasing (Skill 2.E)

In a homework assignment, students will start out with a sign chart to determine the intervals where a function is increasing or decreasing and then write their final answer explaining their reasoning in words. Students will have to use precise language to write their explanations. For instance, if the derivative is negative, the function is decreasing, not decreasing at a negative rate.

5.4 Using the First Derivative Test to Determine Relative (Local) Extrema (Skill 3.D)

In a homework assignment, students will use the first derivative test to determine relative extrema in various representations such as functions, tables, and graphs. Students will use a flow chart in selecting a method for finding relative extrema. The first part of the assignment will have students fill in the blanks to make a true statement. The fill in the blanks will help them develop notational fluency for justifying their answers. In part two, students will apply the flow chart for finding extrema to a variety of settings.

- ☐ Complete Personal Progress Check MCQ A for Unit 5

5.5 Using the Candidates Test to Determine Absolute (Global) Extrema (Skill 1.E)

In a homework assignment, students will use a graphic organizer in selecting a procedure to find absolute extrema. If EVT applies, then students will use the Closed Candidates Test. This homework assignment will include problems of various settings. Students will be expected to identify Global Extrema using functions, graphs, tables, and word problems from prior free-response questions.

5.6 Determine Concavity of Functions over Their Domains (Skill 2.E)

In a homework assignment, students will use a sign chart to determine the intervals where a function is concave up, concave down, and points of inflection. Students are expected to write complete reasons for their answers. For instance, identifying concave up as $f'' > 0$ implies f' is increasing. Students will determine concavity in various contexts using functions, graphs, and tables.

- ☐ Complete Personal Progress Check FRQ A for Unit 5

5.7 Using the Second Derivative Test to Determine Extrema (Skill 3.D)

In a homework assignment, students will use the second derivative test to justify extrema. Students will use the flow chart from the previous topic to help them choose a procedure for determining extrema. What to look for? Students often forget that when they are only given dy/dx and not the original function, they need to consider the second derivative test for finding extrema and not the first derivative test. This kind of problem is addressed in topic 5.12.

5.8 Sketching Graphs of Functions and Their Derivatives (Skill 2.D)

In a homework assignment, students will be given graphs of several functions (polynomials, functions with corner points, and functions with vertical asymptotes) and will sketch their first derivative and in some cases second derivative. CR4

☐ Complete Personal Progress Check MCQ B for Unit 5

5.9 Connecting a Function, Its First Derivative, and Its Second Derivative (Skill 2.D)

In a homework assignment, students will first sketch a function by using the first derivative to determine the intervals where the graph of the function is increasing, decreasing, and relative extrema, in addition to, using the second derivative to determine the intervals where the graph of the function is concave up, concave down, and points of inflection and justify their answers for each. Secondly, students will use their graphing calculator to confirm that their graph is correct. Third, once students verify their graph is correct, they will use their graph of the function to graph the first derivative. Next, students will graph the second derivative from the graph of the first derivative. Students will align all three graphs under each other, i.e., f, f', and f''. Students will then draw vertical lines connecting the following relationships: horizontal tangents on the graph of f is where the first derivative is zero, the relative extrema on the graph of f are points where the graph of the first derivative changes signs, the points of inflection on the graph of f line up with the relative extrema on the graph of f' which supports where the graph of f'' changes signs.

5.10 Introduction to Optimization Problems (Skill 2.A)

Students will complete a home activity. Each student will get one sheet of 8.5 by 11 inch paper. The student has to decide what size square to remove from each corner to produce the greatest volume. The student will remove that size square from each corner, fold up all the sides, and tape the edges to form a rectangular box. This activity is very hands-on and will help students make the transfer to more challenging optimization problems as far as what is the ultimate goal of solving optimization problems. The next day in class, students will learn how to use calculus to arrive at the answer.

5.11 Solving Optimization Problems (Skill 3.F)

In a homework assignment students will learn a 5-step process for solving optimization problems. Students will download a set of cards consisting of

A word problem, picture, and various equations. The student will separate the cards into a group. A group will require a problem statement, a diagram, an optimization equation, a constraint equation, and an optimization function. Then, the students will complete the problems using the steps listed in the chart. Something to watch for is that many students fail to justify their answers using information about the derivative.
If students don't know the closed interval they are working on, then EVT cannot be applied; hence, they should try using a global argument for justifying their absolute maximum or absolute minimum.

5.12 Exploring Behaviors of Implicit Relations (Skills 1.E and 3.E)

This topic will be covered back in topics 5.5 and 5.7. In a homework assignment, students will be given problems where they will have to find absolute extrema given an equation with dy/dx.

- ☐ Complete Personal Progress Checks MCQ C and FRQ B for Unit 5
- ☐ Take Unit 5 Test

Unit 6: Integration and Accumulation of Change (Big Ideas: Change, Limits, Analysis of Functions)

6.1 Exploring Accumulations of Change (Skill 4.B)

The Zoom session will start with a velocity vs. time graph, where $v(t) = 70$ miles per hour from $t = 0$ to $t = 2$ hours.

From that information, students will easily know that the total distance traveled is 140 miles. What is important here is for the students to understand that 140 corresponds to the area under the graph of $v(t) = 70$ from $t = 0$ to $t = 2$ hours. I will also reinforce that as time increases from 0 to 2, that the area under the graph is accumulating more distance because $v(t)$ remains positive. It is also not surprising that the unit belonging to the answer of 140 is miles because when multiplying the units of $v(t)$ and t you get miles. (Note: These are also the units belonging to the dimensions of the rectangle.) I emphasize to my students that we started with velocity measured in mph and ended up with distance measured in miles, so one might say that we are using an inverse process to differentiation. This is where I will show my students how to represent this scenario as a definite integral. Once I establish this problem using definite integral notation, my story continues with $v(t) = -70$ mph from $t = 2$ to $t = 4$ hours. Once students understand that whenever the area of a region bounded by $v(t)$ and the t-axis lies beneath the t-axis, the area represents a negative change in position over the given time, I will ask my students to evaluate many different integrals such as $\int_0^4 v(t)\,dt$, $\int_0^4 |v(t)|\,dt$, using geometry/area and interpret their answer in the context of the problem. In this short practical problem, I am able to build notational fluency with the definite integral, determine the units belonging to the answer of the definite integral, interpret the definite integral in the context of the problem using units, and understand the definite integral as an accumulation function.

6.2 Approximating Areas with Riemann Sums (Skill 1.F)

In a homework assignment, students will approximate the definite integral using Left, Right, and Midpoint Riemann Sums. Problems will be broken into parts consisting of functions, tables, and graphs.

6.3 Riemann sums, Summation Notation, and Definite Integral Notation (Skill 2.C)

In a Zoom session, students will learn how to use the definition of the definite integral to find the exact value of a definite integral. The session will begin with a discussion about how we could improve our approximations from last night's homework in topic 6.2.
Hopefully, a student will respond by increasing the number of rectangles. I will continue this discussion by asking them how many rectangles should we increase it to, and hopefully someone will say an infinite number of rectangles. Then, I will use technology (***Calculus In Motion*** by Audrey Weeks) that will show by increasing the number of intervals, the limit of the sum of the areas of the rectangles approaches a finite number.

Once the definition of the definite integral is established, students will complete a homework assignment finding the exact value of the definite integral using the definition. Then, they will have to write a definite integral given the limit expression representing the definition of the definite integral.

☐ Complete Personal Progress Check FRQs B for Unit 6

6.4 The Fundamental Theorem of Calculus and Accumulation Functions (Skill 1.D)

For a homework assignment, students will complete the activity on Integral-Defined Functions. The activity indicates three different integral-defined functions using the graph of $f(t)$. Students will use geometry/area to find each value in the table. Then, students will graph the values for each function in the table on the grid provided. The students will take away that, although the three functions have a different lower limit as a constant, their rates of change are all the same and that the derivative of each of the integral-defined functions is the graph of f (regardless of the lower limit). Another take-away from the graph is that functions can have the same rate of change yet differ by a constant.

6.5 Interpreting the Behavior of Accumulation Functions Involving Area (Skill 2.D)

As a follow-up to topic 6.4, students will complete an activity in homework applying the Fundamental Theorem of Calculus to understanding the behavior of an integral-defined function. If $g(x) = \int_a^x f(t)dt$, then students will use the connections $g' = f$ and $g'' = f'$ to answer where the function g is increasing, decreasing, concave up, concave down, as well as relative extrema and points of inflection. The vocabulary students used back in Unit 5 to describe the behavior of a function is the same vocabulary they will use to describe the behavior of an integral-defined function.

6.6 Applying Properties of Definite Integrals (Skill 3.D)

Students will complete a homework assignment applying Properties of Definite Integrals. This assignment will consist of various parts. In part $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$, one, students will be given a graph of a function *f* and a definite integral value for a function *g*, and constant function Students will find the value of the given definite integral such as $\int_1^3 (2f(x)+3g(x)-h(x))dx$ using definite integral properties such as $\int_a^b f(x) + \int_b^c f(x)dx = \int_a^c f(x)dx$, and $\int_a^b f(x)dx = -\int_b^a f(x)dx$. Students will have to use area to find the value of the definite integral for *f*. In part two, students will be given multiple-choice questions from prior exams or the item bank where students have to apply these properties. CR6

- ☐ Complete Personal Progress Checks MCQ A for Unit 6

6.7 The Fundamental Theorem of Calculus and Definite Integrals (Skill 3.D)

In a homework assignment, students will use both parts of the Fundamental Theorem of Calculus (FTC) to solve problems. In part one, students will complete the statement using either part of the Fundamental Theorem of Calculus. These are great problems to help students establish appropriate notation in applying the FTC. Students often have difficulty connecting the FTC to problems when the notation varies. In part two, students will use the evaluation part of the FTC to find missing function values; some problems will require technology. In part three, students will be given a graph of a function and will have to use area using either part of FTC.

6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation (Skill 4.C)

The homework assignment will start by having students come up with a position function for an object whose velocity is given by $v(t) = 3t$. This class discussion will generate a rule (general power rule) for finding antiderivatives of the form $\int u^n du$, where $n \neq -1$. Students will often have to perform algebra first before applying this rule. CR6

6.9 Integrating Using Substitution (Skill 1.E)

Students will be given an indefinite integral where they have to recognize the integrand involves the chain rule. A class discussion will include how we could come up with a function whose derivative is in the integrand. This discussion will lead to integration by u-substitution. To help students, they will learn a 3-step process. Students will use the two activities from the ***Teaching and Assessing Modules*** to help them with sequencing the steps in the u-substitution process and error analysis. For each indefinite rule the students learn, the following day they will apply the FTC to evaluating the definite integral using that new rule. Students will learn to use the definite integral feature on their calculator to verify their answer to the definite integral. CR7

- ☐ Complete Personal Progress Check FRQs A for Unit 6

6.10 Integrating Functions using Long Division and Completing the Square (Skill 1.E)

In a homework assignment, students will first have to use Long Division or Completing the Square before finding the antiderivative.

Topics 6.11, 6.12, and 6.13 are BC-only.

6.14 Selecting Techniques for Antidifferentiation (Skill 1.C)

In an activity similar to how the students had to select procedures for derivatives, they will also learn to select techniques for antidifferentiation. In a homework assignment, students will use the strategy of error analysis. Written on the whiteboard of the Zoom Session there will be six integration problems broken into stations. Students will volunteer to discuss what the error is in the integration process and then redo the problem correcting the error and see if the class agrees upon their solutions.

- ☐ Complete Personal Progress Check MCQ B for Unit 6
- ☐ Take Unit 6 Test

Unit 7: Differential Equations (Big Idea: Analysis of Functions)

7.1 Modeling Situations with Differential Equations (Skill 2.C)

In a homework assignment, students will write a differential equation to model a given situation. Prior to the homework, a variety of examples will be demonstrated in class using a chart. In the chart, verbal statements will be associated with mathematical notation; e.g., "the rate of change of y with respect to t is proportional to the amount of y" would be associated with, "$\frac{dy}{dt} = Ky$." Once students know the proper notation, writing a differential equation to model a situation becomes easier for them.

7.2 Verifying Solutions to Differential Equations (Skill 3.G)

In a homework assignment, students will find first or second derivatives to verify solutions to differential equations. CR4

7.3 Sketching Using Slope Fields (Skill 2.C)

7.4 Reasoning Using Slope Fields (Skill 4.D)

Topic 7.5 is BC-only.

7.6 Finding General Solutions Using Separation of Variables (Skill 1.E)

7.7 Finding Particular Solutions Using Initial conditions of Separation of Variables (Skill 1.E)

I put 7.3, 7.4, 7.6, and 7.7 together in one lesson, revisiting the same topics in different contexts for a few days. In a homework assignment, I will start by having students sketch a slope field for a differential equation in terms of just x. By having them sketch the slope field first, they get a feel for what is meant by a slope field while learning how to draw directed line segments that represent the slope of the curve at that point. Also, by starting with a differential equation of just x, students will notice that all the directed line segments are the same vertically, and that if the differential equation was just in terms of y, then all the directed line segments would be the same horizontally. After students construct a slope field, I will give them several points through which they will draw particular solutions and talk about the family of functions that are represented in the slope field. Students start to reason that if the differential equation just had an x then the family of curves (the general solutions) would be represented by parabolas or if the differential equation just had an x^2, then the family of curves (the general solutions) would be represented by the shape of x^3. By starting with these basic differential equations, students are able to make connections from the differential equation → to its slope field → to its general solution → and then finally to its particular solution.

The next type of differential equation my students will work with involves x and y. They will do a matching activity between the slope field and the differential equation. This is where students will have to increase their reasoning with slope fields because they will be looking for where the slope does not exist (which implies vertical tangents), where the slope is zero (which implies horizontal tangents), and where the slopes are positive or negative. Then, we will construct slope fields given a differential equation in terms of x and y and find its particular solution through a given point. A particular example I consider every year is the differential equation given by $\frac{dy}{dt} = -\frac{x}{y}$. Usually, students have no issue with the constructing of the slope field, but they make an error when they draw the particular solution through the point (0, 2) (0, 2) because they make a full circle. Students need to be reminded about particular solutions to differential equations. Particular solutions are functions (that are unique) and are continuous under appropriate conditions. Therefore, the graph of this particular solution belonging to the differential equation above is a semicircle (a function of x) of radius 2 with holes on the x – axis at (-2, 0) and (2, 0). The domain belonging to the particular solution equation must also satisfy the differential equation. The domain for the particular solution, $y = \sqrt{4 - x^2}$ satisfying the point (0, 2) and $\frac{dy}{dt} = -\frac{x}{y}$ is (-2, 2).

As a follow-up to these types of problems, students will be assigned released free- response questions from the College Board website.

☐ Complete Personal Progress Check FRQs A and B for Unit 7

7.8 Exponential Models with Differential Equations (Skill 3.G)

As an extension to topics 7.1 through 7.7, students will not only write a differential equation based on the written problem but will have to solve the differential equation using the initial condition. Some solutions to differential equations will be modeled by exponential functions while others will not. Students often struggle with solving differential equations when there is context. Therefore, to help students transfer what they have learned so far to solving free-response questions like 2011 problem 5, I have created a worksheet for homework that will scaffold the skills necessary to solve such problems. The homework will start by having students practice separating the variables, which is essential for students in order to solve the differential equation.

Then, students will find the particular solutions to differential equations in terms of variables other than x and y. (Students need practice with other variables.) Finally, students will solve the differential equation by modeling a particular solution. Some of these problems will be former free-response questions.

Topic 7.9 is BC-only.

- ☐ Complete Personal Progress Check MCQ for Unit 7
- ☐ Take Unit 7 Test

Unit 8: Application of Integration

8.1 Finding the Average Value of a Function on an Interval (Skill 1.E)

In a lecture discussion, I will show the students how to derive the formula for finding the average value of a function on a closed interval. Students will be assigned a homework assignment where they will have to find the average value of a function in multiple representations including functions, graphs, and written contexts. CR4

8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals (Skill 1.D)

In a homework assignment, students will apply the Fundamental Theorem of Calculus to problems pertaining to position, velocity, and acceleration. In some problems, students will be given information about the velocity of an object in multiple representations and will answer questions about the object's position. In other problems, students will be given information about the acceleration of an object, the object's velocity and position at a time t, and will be asked to find particular solutions for the object's velocity and position.

8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts (Skill 3.D)

This can be a challenging topic for my students because they have not internalized some concepts nor linked them to the appropriate notation. For example, some questions are asking for an average rate of change while other questions are asking for the average value. Because these questions seem very similar to each other, students are not sure what notation to write. These types of questions, usually referred to as "Rate Questions," involve rate of change, average rate of change, average value, find the amount of a quantity given the rates of change, etc. In a class activity, I will use a flowchart to help my students learn how to set up the integral. In the activity, students will write what functions they are given in the middle of the flowchart and label the units. The units and notation mean everything and cannot be ignored. Working from the middle of the flowchart, students will work upwards, interpreting various integral expressions using units and context; while working downwards, students will interpret derivatives using units and context. Some problems will require the graphing calculator for both derivatives and integrals. CR7

- ☐ Complete Personal Progress Check FRQ A for Unit 8

Topics 8.4 through 8.6 Area Between Curves
(Topic 8.4, Skill 1.E; Topic 8.5, Skill 1.E; Topic 8.6, Skill 2.B)

Students will complete a homework assignment where they will find the area between two or more curves. Throughout these topics, students will have to find areas between functions of x and as well as areas between functions of y. Some problems will have more than two points of intersection Some problems will require the students to use their graphing calculator's equation solve to find the points of intersection, then use the calculator to graph the functions to determine which graph is higher and finally use the calculator's numerical integration feature to approximate the area. CR7

- ☐ Complete Personal Progress Check MCQ A for Unit 8

Topic 8.7 and Topic 8.8 Volumes with Cross Sections
(Topic 8.7, Skill 3.D; Topic 8.8 Skill 3.D)

Students will complete a homework assignment where the base of a solid is the region enclosed by given equations and the known cross sections consist of either a square, rectangle, triangle, or semicircle.

To help students visualize what these solids look like, I will use the software – Calculus In Motion.

Topics 8.9 through 8.12
(Topic 8.9, Skill 3.D; Topic 8.10, Skill 2.D; Topic 8.11, Skill 4.E; Topic 8.12, Skill 2.D)

In a class lecture, students will find out about volumes of revolutions by using the washer and disk methods. I will start the lesson with an enclosed region bounded $\sqrt{x}$ by the graph of $y =$ and the $x -$ axis from $x = 0$ to $x = 4$. Next, using Calculus In Motion, the software will revolve one rectangle about the $x -$ axis showing the result of one cross section – a disk. Using the volume formula for a cylinder, I will derive in a class lecture a formula for finding the volume of revolution by disk. I will reinforce to students how important it is to draw the rectangle in the enclosed region. The height of the rectangle represents the length of the radius and when the rectangle is drawn vertically, the length of the rectangle is the top function minus the bottom function resulting in an integrand in terms of x.; and when the rectangle is drawn horizontally, the length of the rectangle is the right function minus the left function where integrand is in terms of y. After a few examples and a homework assignment, I will return to this same problem (same region bounded by the $x -$ axis) the next day where the axis of rotation gets dropped to the line $y = -2$. Using a downloadable *Calculus in Motion*, I will revolve one rectangle about the line $y = -2$, and the students will see that the result of one cross section is a washer. Using a discussion, I will derive with the students a formula for finding the volume by the washer method. To help students with setting up the formula, I will require students to draw the rectangle perpendicular to the axis of rotation; this way, the students will be able to recognize the big radius (representing the radius of the outer cylinder) and the clear radius (representing the radius of the inner cylinder). Volume by washer is recognized in my class by the formula

$$V_W = V_{big\ radius^2} - V_{clear\ radius^2}$$

Once students learn all the volumes of revolution methods, they often get confused on how to set up the integral. To help my students learn the set-up of volumes by disk and washer, we will complete a chart of problems that allows them to make the connection between the picture (the enclosed region and axis of rotation) and the appropriate volume method.

After Topic 8.9, students may complete Personal Progress Check FRQ B for Unit 8.

Topic 8.13 is BC-only.

- ☐ Complete Personal Progress Check MCQ B for Unit 8
- ☐ Take Unit 8 Test

Course Objectives

At the end of the course, students should be able to solve a variety of real-world problems using limits, derivatives, integrals, and series. Students are shown the interrelationships of these four major themes/threads throughout the course. The course teaches the students how to communicate their mathematical reasoning using proper mathematical terminology in complete sentences. Students are instructed how to answer problems in the context of the problem, both verbally and in written sentences/paragraphs, using appropriate measurement units.

Prerequisites

All students who are taking AP Calculus BC have completed precalculus and have a firm understanding of:

- Functions – their graphs and behaviors
- Trigonometry
- Logs and Natural Logs
- Transformations and Translations
- The use of their graphing calculator to solve problems
- The value of the Rule of Four to solve problems (analytical/algebraic, numerical,
- graphical, verbal/communication)
- Transcendental Functions

These and other prerequisite topics/skills are briefly reviewed, as needed, during the year to help students make valuable connections between the big ideas.

Assessment

Students are assessed using several methods. I count the daily homework which is submitted as a scanned PDF file to their DropBox as 10% of a student's grade. The other 90% is a combination of quizzes, labs, projects, Discussion Board participation, and unit tests. I will use the Personal Progress Checks (PPCs) designed by the College Board as formative assessments during the course of the 10 units to help students and me better understand what concepts my students are struggling with. The unit tests contain a no calculator section and a calculator section consistent with the AP Calculus BC Exam. Weekly labs and discussions consist of graphical, numerical, and analytical components and a written conclusion as a journal entry. Free-response questions are graded similar to the AP Exam. A midyear exam is given at the end of the first semester. Just before the AP Exam in May, students are given an entire AP BC Calculus practice exam, which is graded like the actual exam using the scoring guidelines published by the College Board. This is counted as their final exam grade for the year.

Because the mathematical communication component is so important in this class, students are strongly encouraged to do test corrections for every exam and scan them into their DropBox. These test corrections are an integral component of the learning process for this AP course and will help students understand the required concepts, as well as how to effectively communicate their answers.

Review for AP Exam (2–3 weeks)

- Complete review of the proceeding major topics/concepts.
- Students work through problems by using published review books and College Board released MC, FRQs, and practice exams.
- Nightly practice problems from released AP Exams, including both multiple- choice and free-response questions.
- A complete practice exam (most recently released public exam) given over several class periods. This exam is graded just like the AP Exam using released scoring rubrics.

Teaching Strategies

One of the major outcomes of this course is for students to be able to work with functions represented in a variety of ways: graphically, numerically, analytically, and verbally. This is accomplished in a variety of methods, including bi-weekly homework presentations by students on the Discussion Board, downloadable problem sets with written justifications using correct mathematical nomenclature.

All exams are modeled after the AP Exam, including multiple-choice and free-response questions. All exams have both a calculator and a no calculator active section. Students are taught to round once during a problem, at the end of the problem and to three decimal places. Students are taught to develop connections/relationships between the three major themes of the course: change/limits/analysis of functions. These three themes are woven throughout the course as described in the CED. Students use their calculators to explore concepts and solidify their learning experiences. Students are exposed to a multitude of problems from various textbooks, review books, and other sources.

Application of Calculus Project:

Students will complete a project pertaining to optimization. Students will come up with a product that they want to manufacture and decide on how much of that product they want to package and sell. Students will provide in detail how much it will cost for them to manufacture and package their product. Then, students will use calculus to determine the dimensions of their packaging that will determine the least amount of material used to package their product. Finally, students will use all of their information to determine how much they should sell their product for to make their desirable profit.

Additional Activities/Projects

1. The students are given two complicated functions expressed analytically that represent the rate of change of the populations of wolves and coyotes. They must first use the calculator to draw the graphs, then use the calculator's equation solver to find where the graphs intersect, and third use the calculator's numerical differentiation feature to estimate the slope of each graph at the intersection point. Lastly, they must set-up definite integrals to find the net increase or decrease in each population over a given time period and solve with the calculator's numerical integration feature. CR7
2. The students are asked to graph various functions and their derivatives on the calculator in order to explore the relationship between the graph of the function and the graph of its derivative and discover any useful connections (such as that the graph of f' is increasing where the graph of f is concave up). CR7

3 The students are asked to deal with a real-world economic situation confronting complex functions dealing with actual optimization issues.

Tesla Production Optimization

Scenario: Tesla's Fremont factory produces Model 3 vehicles. Production data shows:
- Production rate: $P(t) = 1000 + 200\sin(\pi t/6)$ cars per month (t in months)
- Production costs: $C'(x) = 45{,}000 - 15x + 0.002x^2$ dollars per car
- Market demand follows: $D(t) = 1200 - 50e^{(-0.1t)}$ cars per month

Use of calculator for Individual Analysis:

1. *Integration Application:* Find total cars produced in first year
2. *Optimization:* Determine monthly production level that minimizes average cost per car

Economic Decision: Recommend optimal production strategy based on mathematical analysis.

Integration Application - Total Cars Produced in First Year
Calculator Setup:
Find $\int_0^{12} P(t)\,dt = \int_0^{12} [1000 + 200\sin(\pi t/6)]\,dt$
Solution: $\int_0^{12} [1000 + 200\sin(\pi t/6)]\,dt$

$= [1000t + 200 \cdot (-6/\pi)\cos(\pi t/6)]_0^{12}$

$= [1000t - (1200/\pi)\cos(\pi t/6)]_0^{12}$
$= [1000(12) - (1200/\pi)\cos(2\pi)] - [1000(0) - (1200/\pi)\cos(0)]$
$= [12000 - (1200/\pi)(1)] - [0 - (1200/\pi)(1)]$
$= 12000 - 1200/\pi + 1200/\pi$
$=$ **12,000 cars**

Answer: Tesla produces exactly 12,000 cars in the first year.

Optimization Application - Minimize Average Cost Per Car
Calculator Setup:
Given: $C'(x) = 45{,}000 - 15x + 0.002x^2$
First find total cost function: $C(x) = \int(45{,}000 - 15x + 0.002x^2)dx$
$C(x) = 45{,}000x - 7.5x^2 + (0.002/3)x^3 + K\ C(x)$
$= 45{,}000x - 7.5x^2 + 0.000667x^3 + K$
Average Cost Function: $AC(x) = C(x)/x = 45{,}000 - 7.5x + 0.000667x^2$
Minimize Average Cost: $AC'(x) = -7.5 + 0.001333x = 0$
Solving: $x = 7.5/0.001333$
= **5,625 cars per month**
Verification (Second Derivative Test): $AC''(x) = 0.001333 > 0$ ✓ (confirms minimum)
Minimum Average Cost: $AC(5625) = 45{,}000 - 7.5(5625) + 0.000667(5625)^2$
$= 45{,}000 - 42{,}187.5 + 21{,}093.75$
= **$23,906.25 per car** CR7 CR8

4. In this calculator-active, Zoom break-out session, students will become familiar with the graphing of trigonometric and rational functions in a real-world context. Student will find the area bound by trigonometric curves and volume of curves using cross sections to the x-axis. Students will integrate both trigonometric functions and rational functions in order to find zeros for displacement, maximums, minimums, and average values. These values will be presented with explanations to address specific questions concerning the cooling of a house, total cost of cooling a house over a particular day, number of people entering a park, dollars collected for admission to a park, and predicting when the number of people in the park is a maximum. CR8